Lecture 12: 10/01/2018

Synchrotron Radiation (Cont'd):

Synchrotron radiation from astrophysical sources is typically produced by an ensemble of particles. This removes the pitch angle from the expression for power, assuming an isotropical distribution, and one has to consider an energy distribution for the electrons. The total power radiated by an ensemble as a function of frequency is given by:

$$P_{tot}(\nu) = \int_{E_{min}}^{E_{max}} P_{(\nu)} N_{(E)} dE$$

Here $N_{(E)}$ is the energy distribution function and E_{min}, E_{max} are the minimum and maximum energy of the particles in the distribution respectively. Here, we consider synchrotron radiation from thermal and non-thermal ensembles both.

Thermal Synchrotron Radiation:

In this case, assuming that $E \gg m_e c^2$, we have:

$$N(E) \, dE = N_0 E^2 \exp\left(-\frac{E}{kT}\right) dE$$

This results in:

$$P_{tot} (\nu) \simeq \frac{\sqrt{3} \, e^3 n_e B}{8\pi m_e c^2} \left(\frac{\nu}{\nu_T}\right) I\left(\frac{\nu}{\nu_T}\right)$$

Here, n_e is the electron number density and:

$$\nu_T \equiv \frac{3 e B (kT)^2}{4\pi m_e^3 c^5} \quad , \quad I_{(n)} \equiv \frac{1}{n} \int_0^\infty u^2 e^{-u} F\left(\frac{n}{u^2}\right) du$$

Where:

$$F_{(y)} \equiv y \int_y^\infty K_{\frac{5}{3}}(u) \, du$$

Much of the calculational effort in determining $P_{tot}(\nu)$ goes into the evaluation of $I\left(\frac{\nu}{\nu_T}\right)$. This function can be approximated as a power law over most of its range, although with different indices above and below $\nu \sim 1$. It is seen that the spectrum is peaked around ν_T.

In general, therefore, thermal processes are not very efficient unless the plasma temperature is very high ($kT \gg m_e c^2$). Hence, in most cases, thermal synchrotron sources are too faint to be seen easily. This situation has changed in recent years as the instrument sensitivity has continued to improve. For example, we will later talk about the supermassive black hole at the center of the galaxy. This is an excellent example of an object whose spectrum includes a thermal synchrotron component, which has emerged as an important high energy source in recent years.

Non-thermal Synchrotron Radiation:

Most synchrotron sources, including the relativistic jets in AGNs and the shells in supernova remnants, are non-thermal sources. In these sources, electrons are

accelerated to relativistic velocities. Their distribution

is given by a power law in this case:

$$N(E) \, dE = K E^{-\eta} \, dE$$

Here, K is a normalization constant and the spectral index

η typically varies over the range $\sim 2-2.5$.

As we mentioned, for a given energy E, most of the

radiated power is at $\nu_c(E)$:

$$\nu_c(E) \simeq \left(\frac{E}{m_e c^2}\right)^2 \nu_{gyr}$$

Thus:

$$dE \simeq \frac{m_e c^2}{2(\nu_{gyr} \nu)^{1/2}} \, d\nu$$

Here, we have dropped the subscript "c" on ν_c. Recall $^{that:}$

$$P_{sync} = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B \simeq \frac{4}{3} \sigma_T c \left(\frac{E}{m_e c^2}\right)^2 \frac{B^2}{8\pi} \qquad (\beta \simeq 1)$$

We then find:

$$P_{tot}(\nu) = \int P_{sync} \, N(E) \, dE \propto B^{\frac{1+\eta}{2}} \nu^{\frac{1-\eta}{2}}$$

A precise calculation yields the complete expression in below;

$$\rho_{tot}(\nu) = 1.7 \times 10^{-21} a_{(n)} K B^{\frac{1+n}{2}} \left(\frac{6.26 \times 10^{18} Hz}{\nu}\right)^{\frac{n-1}{2}} erg \ cm^{-3} s^{-1} Hz^{-1}$$

Here, $a_{(n)}$ is a function that depends only weakly on the spectral index n.

The most important property of non-thermal synchrotron radiation is the power-law spectrum with index $\frac{n-1}{2}$. However, one should note that it is not valid over all frequencies. Indeed, the spectrum is divergent at low frequencies. But exceeding the blackbody limit is not permitted on physical grounds. In fact, thermodynamic considerations tell us that the emissivity of a plasma cannot exceed the limit imposed by the maximal rate at which the plasma can absorb energy, which basically constitutes a blackbody limit.

A quick solution to the divergence of $P_{tot}(\nu)$ at low
frequencies would be that $\overset{\text{the}}{\wedge}$ non-thermal synchrotron
spectrum should be a power law except below some
break frequency ν_b. The spectrum would go $\propto \nu^2 T$
below ν_b, similar to the Rayleigh-Jeans portion
of the blackbody spectrum. Here, T is an "effective"
temperature, which can be frequency dependent rather
than an actual temperature that exists for a thermal
distribution. This "effective" temperature is related to
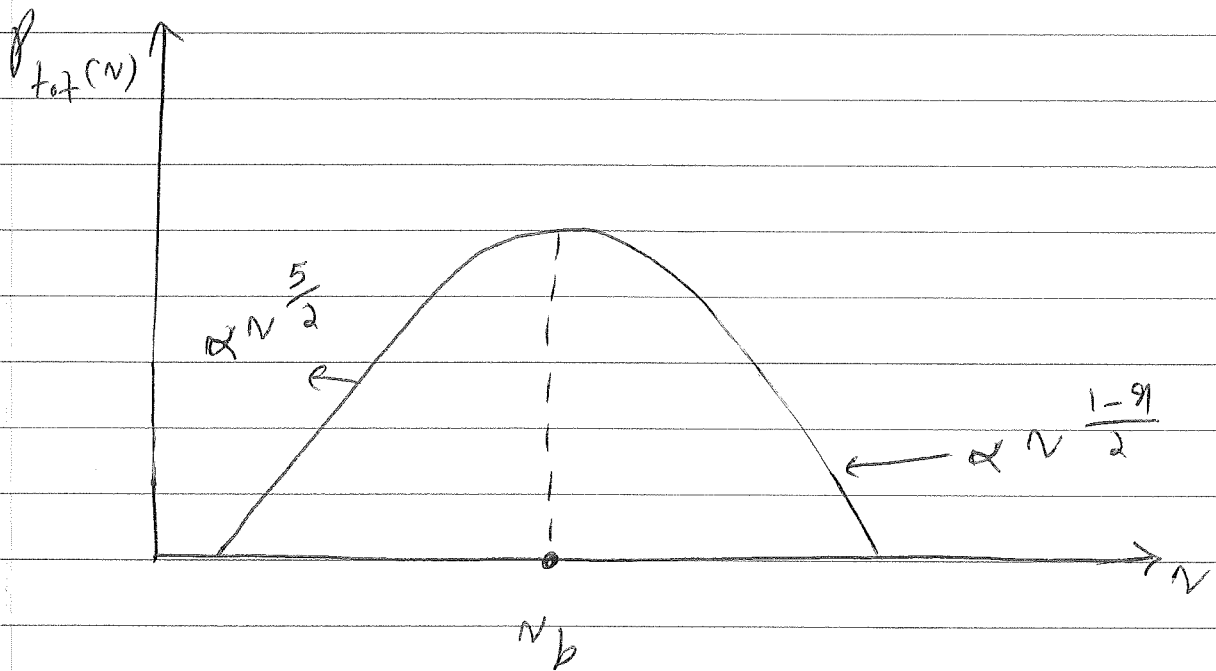the energy of the electrons, which results in:

$$\gamma m_e c^2 \doteq \frac{3}{2} k T_{(\nu)} \Rightarrow T_{(\nu)} \sim \frac{m_e c^2}{3k} \gamma \Rightarrow T_{(\nu)} \sim \frac{m_e c^2}{3k} \left(\frac{\nu}{\nu_{gyr}}\right)^{\frac{1}{2}}$$

The synchrothron spectrum below ν_b therefore is:

$$P_{tot}(\nu) \propto \nu^{\frac{5}{2}}$$

In consequence, the non-thermal synchrotron spectrum looks

as follows:

$P_{tot}(\nu)$ plotted against ν, rising as $\propto \nu^{\frac{5}{2}}$ at low frequencies, peaking near ν_b, and falling as $\propto \nu^{\frac{1-q}{2}}$ at high frequencies.

This spectrum is unique and informative. The $\nu^{\frac{5}{2}}$ behavior at low frequencies is a clear indication that the radiation is non-thermal. The slope at high frequencies (in log scale) provides us with the spectral index of the emitting particles. Finally, the break frequency ν_b is a measure of the optical depth through emitter. Moreover, the expression for $P_{tot}(\nu)$ may be used to estimate the magnetic field B.